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The steel supply chain generally comprises a steel producer, manufacturing semifinished products, and a steel user, manufacturing formed components. A review is presented of the deformation models needed by the steel industry, considering the current and expected future requirements of the end users. A trend that is expected to accelerate is their increased use of automation. This imposes a tighter dimensional tolerance, which requires, among other things, models that predict rolling loads and physically based models being introduced, offering distinctive benefits that will be discussed. A narrow distribution of mechanical properties is needed, requiring techniques that link deformation parameters such as strain, time and temperature, through microstructure models to property prediction, and this should include toughness as well as yield strength. Being able to predict toughness at all points in a rolled product would significantly reduce waste and costs associated with destructive testing.

The need for lightweight designs is leading to wider application of high-strength steels. Because of the requirements of lower cost and good weldability, the high strengths are being achieved through leaner chemistries, using controlled thermomechanical processing, where the transformation from austenite needs to be incorporated into the models. This is a major difference between steel and aluminium.

Laser fabrication techniques are increasingly being used; these require high degrees of flatness in the steel, including after it has been cut, where non-uniform residual stress distribution can lead to distortion. Models that predict residual stresses are needed, and, since the customer might buy the steel in a coil and uncoil it himself, the residual stress model and data will need to be shared. Customers will become more than recipients of steel; they will share in the knowledge embedded in the steel, and the ability to do this, along the varying deformation processes used by the steel industry, is a challenge for those developing the modelling techniques.

> Keywords: deformation modelling; finite element; process control; hot rolling; steel; through-process model

### 1. Introduction

The steel industry in the UK, probably in part because of the presence of large companies such as British Steel, is traditionally viewed as a primary metal manufacturing industry. With this perspective, the deformation models needed would be those that made the processes used, such as hot rolling, more efficient: producing better quality steel at lower cost. However, the steel industry is probably better viewed as a supply chain, comprising, for example, a steel producer manufacturing semi-finished product, a processor forming steel parts, and an equipment manufacturer, assembling

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Figure 1. (a) A three-dimensional perspective of a coupled model of the rolling of a steel section. Because of symmetry, only the top half section is modelled. (b) End-on view of the local strains during one of the pass sequences shown in (a).

steel parts, perhaps with those from other materials, maybe including a machining operation. Although it remains vitally important to improve the modelling capability of the deformation processes used in making semi-finished steel products, it will be argued that we must, in developing these models, do so bearing in mind that in the future they should have the capability of transferring data and knowledge along the supply chain.

In this paper, the deformation-modelling needs of this steel-industry supply chain will also be discussed from the point of view of the current and expected future requirements of the end user or customer. The reason for this choice is that if the models add value to the customer, then they will benefit the industry as a whole. After an overview of examples of deformation models currently used, some trends in the use of steel and their consequence on the development of such models will be described.

### 2. Deformation models used by the steel industry

## (a) Finite-element models (FEMs)

The first FEM used in the steel industry was probably to model the elastic deflection of a component under load. As computing power grew, more sophisticated models became possible, and the plastic deformation during hot rolling of steel, including friction effects at the rolls and temperature variations across the section, became possible. Figure 1a shows the modelling of rolls and workpiece in the rolling of a beam section, and figure 1b shows the locally predicted deformation strains across the section during a single pass. Being able to model the metal flow enables the roll pass sequence to be designed on the computer rather than in a trial and error mode on the plant. As well as enabling the design to be done at lower cost, the understanding of the process, which comes from trying to model it, more readily allows process optimization. Reducing the number of passes needed to go from the initial to the final shape has a significant influence on the cost of the product.

The analysis illustrated in figure 1a was done using a so-called coupled model, where the forces causing the workpiece deformation are allowed to deflect the rolls. Such techniques give increased accuracy of prediction, albeit at the expense of increased computing power.

Currently available finite-element (FE) software will do most of the things the steel industry needs. Future developments in the software will be incremental; however, step changes in the effectiveness of the technique will be brought about by better material data, such as accurate friction coefficients with or without oxide scale, and constitutive models to feed into the FE package, and also by increased computing power to make each run faster. Artificial intelligence systems will be used to short cut a full FE analysis, particularly in designing rolling schedules within a product family, where 'rules' of how the metal flows in a given situation can be built in. This may not be quite as accurate as the full FEM, but could allow significantly faster exploration of 'what if?' scenarios.

Figure 2a shows an FE calculation of the shape resulting from the compression testing of a simple cylinder, and a comparison of the results of full FE calculations with those from a trained neural network are shown in figure 2b, where it can be seen that excellent agreement is obtained. However, the accuracy of the neural network prediction depends on the size of the training data-set, as can be seen in figure 2c, where a linear relationship is observed for large data subsets. If insufficient validation data are used, then the errors can be large, and the prediction method becomes unstable, the linear relation no longer being found. Within the linear range, the trade-off is accuracy versus cost; the more data used to set up the network the more accurate the result.

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Figure 2. (a) An FEM of the plastic strain during a compression test of a cylinder. Because of symmetry, only a quarter of the cross-section is modelled. (b) A comparison of the shape calculated using a full FEM with those of a neural network (NN) showing excellent agreement. (c) The relationship between error in the NN prediction with the size of the data-set used to train and validate the model.

#### (b) Microstructural models

One of the major developments in physical metallurgy over the last 50 years or so has been the understanding of the relationship between the properties of a material and its microstructural characteristics. The relationship between grain size and yield strength, and precipitation hardening theories, all made major contributions. A consequence of this understanding has been a strong drive to include microstructural parameters in the deformation models used. Rather than treating steel at the macroscopic level as a homogeneous solid described by the constitutive equations of a mathematician, efforts have been made to incorporate our knowledge of what happens to the microstructure into the modelling of the deformation of steel. One of the benefits resulting from this approach is the transferability of the model from





Figure 3. (a) The measured grain-size distribution across a bar compared with that calculated (b) from the process parameters. (c) The predicted grain-size distribution after adjusting the pass sequence to achieve a more uniform grain-size distribution.

one particular rolling mill and steel composition to another. Purely adaptive models are specific to the situation for which they were set up.

One factor, important to many customers who buy steel bar, is its machinability. The economics of machining components from a bar can depend critically on a high reproducible machining rate. Although grain size might not be considered a primary parameter in influencing machinability, it can, nevertheless, be important.

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Figure 4. The flow stress at 900 °C as a function of holding time.

One customer experienced a problem with 'chatter': uneven machining around the circumference of a bar. Microstructural examination demonstrated an uneven distribution of grain size (figure 3a). The factors that control grain size are reasonably well understood, so it is possible to include algorithms describing the grain size based on experimentally determined parameters such as deformation strain, time, temperature, etc. Figure 3b shows the prediction of grain size across a bar, inputting the deformation history from the process used to produce the bar. It can be seen that excellent agreement is obtained between the measured and predicted grain size. With this level of understanding, then it is relatively straightforward, on the computer, to make adjustments to the deformation pattern, to induce recrystallization across the whole cross-section. A much more uniform grain-size product was predicted (figure 3c), and this slightly modified roll pass sequence was then implemented, quickly solving the customer's problem.

The evolution of the austenite state has a dramatic effect on the required loads during the rolling process. Incorporating this evolution into a model of the rolling process significantly improves its accuracy. During hot deformation, the grain size is affected by repeated mechanical working and recrystallization. However, if the time between successive deformation passes is too short, the steel does not necessarily recrystallize. This affects not only the final austenite grain size (and, therefore, product properties), but also the hot strength of the steel at the next pass, which influences predictions of roll gaps, spread and also dimensional tolerance. Figure 4 shows the stress measured in a stress relaxation test on a C–Mn steel deformed to a strain of 0.69 at 900  $^{\circ}$ C, plotted as a function of log time.<sup>†</sup> It can be seen that there is a change of slope corresponding to the onset and completion of static recrystallization. Ensuring that there is enough time between deformation passes, to allow recrystallization always to occur, gives significantly reduced dimensional variation in the product, as well as reducing work loads on the roll stands.

† Unpublished research at the University of Sheffield as part of a DTI Link Enhanced Materials Programme, with British Steel and the University of Cambridge as partners.

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Figure 5. A prediction of the ferrite grain size and the prior austenite grain size at different positions in the cross-section of a beam similar to that shown in figure 1b. The measured ferrite grain sizes are shown for the same positions.

So far in the discussion, the microstructural evolution has taken place entirely in the austenitic condition; in other words, the initial deformation temperature and the rate of cooling during rolling never bring the steel into the transformation temperature range until deformation is complete. The final austenite grain size and the ferrite grain size, to which it transforms, can be calculated from a combination of material composition using a 'carbon equivalent' value, rolling parameters, including retained strain, and cooling rates. Figure 5 shows the predicted austenite grain size at different points across the section of a rolled beam, with the predicted ferrite grain size after cooling and transforming, and this is compared with the measured ferrite grain size. It can be seen that, although the predicted austenite grain size varies significantly with position, the predicted ferrite grain size shows much less variation, and that excellent agreement is obtained with the measurements. Adequate models exist to predict the transformation to ferritic-based structures, at least for C–Mn steels, but more work is needed for martensitic- or bainitic-containing microstructures. For cost reduction reasons and for improved weldability, there is a drive to achieve the same property combination with a leaner chemistry: a steel with a lower alloving content. This requires other strengthening methods to be used, and one approach is to finish roll at lower temperatures, where the steel transforms during the deformation process itself. More microstructural models are needed for this thermomechanical rolling, and for situations where the transformation is subsequently to bainite or martensite. In particular, techniques to predict the transformation microstructure and the mechanical properties of the transformed product are needed.

Considerable waste occurs because of the need to take samples for mechanical testing to confirm that the piece of steel has the properties specified by the customer. As well as the time and cost of removing the material, machining the sample and

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doing the test, a relatively large amount of steel might have to be scrapped to take out just a small sample. Clearly, if the steel producer can predict the properties, both tensile and toughness (Charpie) to an accuracy that the customer will accept, then the whole manufacturing process can be made more efficient. Such models should include the property change in forming the final component, for example the increase in strength in forming plate into pipe, and will ultimately include predictions of the properties of the weld.

Another area where our models are currently not sufficiently well developed is that of the dynamics of precipitate dissolution and reprecipitation. We need to be able to predict the grain growth during heating to the hot rolling temperature as some of the precipitates stabilizing the grains dissolve, and, similarly, the inhibiting effect on grain growth as the workpiece cools during rolling and the precipitates reform.

## (c) Constitutive models

The FE packages described in the previous section require information on the stress–strain behaviour of the steel at the temperature and strain rate of the deformation. Thermomechanical simulators can be used, for example, to do hot compression tests at temperatures from  $ca. 1200 \,^{\circ}\text{C}$  down to 700  $^{\circ}\text{C}$  at strain rates typically in the range  $0.1-10 \, \text{s}^{-1}$ . However, during hot rolling of steel, dynamic recrystallization can occur during the deformation, grain growth takes place between rolling passes and a variety of other variables are introduced that are not present in the simulation. Consequently, the approach usually taken is to have some form of viscoplastic constitutive equation defining the flow stress in terms of material constants (a recent paper by Farrugia *et al.* (1998) used 12), and these constants are then identified from material data.

## (d) Crystallographic texture

For many steel products, texture effects are relatively unimportant. They are usually ignored for the deformation models used for the hot rolling of carbon steel sections for example. However, in the grades of ultra-low carbon and interstitial-free strip steels being introduced into the automotive market, they can be important. Figure 6 shows the dependence of the anisotropy factor r, the ratio between longitudinal and transverse strains in metal forming, as a function of the fraction v of grains near the {111} orientation, a measure of texture. The anisotropy influences the formability of these steels, which is a key factor in their application. The data demonstrate that more attention must be given to understanding and predicting texture evolution during deformation and heat treatment for these new strip grades.

#### 3. Trends in the use of steel

A significant recent trend in the application of steel, which is expected to increase in importance in the future, is the use of automatic equipment to handle and process it. Pick and place robots have limited capability of handling variation in a product's dimensions. If a component is made from sheet steel by bending in an automated press, then any variation in yield strength or in thickness of the steel will be reflected in variation in the springback after the bending load is removed, or, in other words, it will result in variation in the bent angle of the formed component. A manual



Figure 6. The anisotropy factor r as a function of the volume of grains near a {111} orientation. The data are for coiling temperatures of 625 °C ( $\circ$ ), 675 °C ( $\circ$ ), 725 °C ( $\Box$ ). Consecutive points for each steel correspond to hold times of 0, 60 and 180 s at 850 °C before coiling.



Figure 7. The modelled austenite grain-size evolution during a typical strip rolling schedule.

assembly operation might be able to adjust for this, a robotic one might not. There is, thus, a strong drive to reduce variation.

The process control models used to operate a rolling mill have mostly been black box or adaptive. A measurement is made of a product parameter, e.g. thickness of a coil, and compared with the requirement. A process parameter, e.g. rolling load, is then adjusted and the cycle repeated until the product requirement is being met within the control limits. However, significant improvements have been made recently by including prediction of the grain size, based on deformation history, and a knowledge of the relationship between grain size and flow stress to predicted rolling

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Figure 8. The error distribution in predicted set-up load for a purely adaptive model compared with microstructural model (MetModel) including adaption.

loads. Figure 7 shows a prediction of the austenite grain size during hot rolling on a strip mill, and figure 8 shows the error in predicted set-up load for many different trials, comparing a purely adaptive 'black-box' model with one that includes the microstructural model. Clearly, the latter gives a much tighter distribution around the zero-error position. Better prediction of rolling load gives less variation in the thickness of the product, exactly the attribute desired by the customer. Even more significant is the fact that the first coil of a new run is used as a 'training' coil by the adaptive model, and might not be within specification, whereas the model incorporating microstructural prediction results in the first coils of a run being within the acceptable dimensional range.

Not having to scrap or downgrade the first coil is a tangible success from a physically based model, and points the way to the future. The quality of products will be improved by using process control models that incorporate as much of our physical metallurgy knowledge as possible. This drive towards physically based models will be facilitated by increasing computer power, enabling the complexity of such models to be addressed directly, rather than having to make expedient simplifying assumptions.

Laser welding is another process increasing in importance because of its speed and reliability. A filler wire is not always used, so this requires a high degree of fit when two components to be welded together are pre-assembled. Any gap or outof-plane mismatch cannot be accommodated. In manufacturing components from sheets of steel, the steel is often sold as a semi-finished coil, which is then uncoiled, flattened and cut to size. Uncontrolled residual stresses in the steel can lead to severe distortion on cutting, as can be seen on the left in figure 9, whereas on the right a similar steel processed to control the residual stress is shown, the pieces remaining flat after cutting. Clearly, unless the strip stays flat after slitting, it cannot be butt welded with lasers without an additional operation.

Figure 10 shows an FEM of the roller-levelling process, where the input material has an edge wave as a consequence of the roll gap not having been parallel in a previous rolling operation. In rolling strip, the metal flow is predominantly longitu-

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Figure 9. The severe distortion on slitting sheet containing non-uniform residual stress (left) with the minimal distortion (right) when the processing is controlled to give uniform residual stress.



Figure 10. The roller-levelling process.

dinal, so that a non-parallel roll gap results in one edge being elongated more than the other and produces the wave. In a roller-leveller, the top set of rolls lies in the 'valleys' of the bottom set, subjecting the steel strip to alternating reverse bending as it passes through. The resulting material on the exit side of the roller-leveller (shown in figure 10), is flat; however, the residual stress represented by the different shading

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Figure 11. Tool and blank set-up for simulation of a press forming process.

contours varies along the length and width of the strip. This could lead to distortion on cutting, when some of the constraints are removed. Obtaining flat sheet, which does not distort during further processing, is potentially an iterative process between varying the rolling conditions (reducing or eliminating the ingoing edge wave in figure 10) and varying the roller-leveller set-up, or the number of roller-leveller passes. Increasing the plastic deformation during the reverse bending results in more uniform residual stress from point to point along the surface. Modelling the residual stresses enables the development of an understanding of the influence of all the variables on the residual stress distribution after roller-levelling, and this iteration can be done on the computer. This information can then be used to process strip with a controlled residual stress, resulting in product that does not distort on slitting.

Steel car-body parts are often made by pressing from sheet steel, and FEMs can be used to simulate the press forming process. Figure 11 shows the tool and blank set-up prior to such a simulation. The local strains experienced during the forming operation can be predicted, and then compared with a formability limit model to evaluate whether such a limit is close, in which case, splitting can potentially occur and the die or material might need to be modified. It is interesting to note that the starting FE mesh in figure 11 is uniform; a homogeneous material is assumed because the knowledge of the actual material condition, for example residual stress distribution from prior processing, is not always available. In the examples shown in figures 10 and 11, the modelling was actually done by British Steel, but in practice, the roller-levelling could be done at an independent steel service centre and the final

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Figure 12. A schematic illustration of an adaptive model for process control.

pressing at an auto supply company. The design of the steel component itself, as well as the process to make it, can be improved if the processing knowledge embedded in the steel can be transferred along the supply chain. In other words, greater accuracy of the forming-limit prediction will be achieved if the effects of prior deformation history can be included. This requires that the models used and the software they run on should be common along the chain; some of the issues in implementing this are discussed in the next section.

#### 4. Through-process model

Deformation models, used for process control, have to run in real time and need to handle minor variability in feedstock composition and temperature. Figure 12 illustrates, schematically, an adaptive process control model, where an output such as thickness is measured, compared with a desired value in the control system, which then pulls some 'lever', changing the process to move the measured value closer to the desired value. Physically based models are often too slow for real time control, but the inexorable increase in affordable computing power might soon allow this. In the meantime, artificial intelligence, such as neural networks, can provide a bridge between a purely adaptive model and a physically based one, as illustrated in figure 13.

So far we have only discussed the control of a single deformation process in isolation. Figure 14 represents two sequential steps, such as a hot rolling mill feeding a

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Figure 13. A schematic illustration of the use of a neural network to apply features of a physically based model to direct process control.

cold mill downstream. We know that even the crude transfer of data between these two processes can result in better control. Running the coils through the cold mill in the same sequence they came off the hot mill results in better final gauge control. Maintaining the same sequence means that any slow drift in any output measure of the hot mill is also seen by the cold mill control as a slow drift. Putting the coils through the cold mill in a random sequence can break down the slow drift into a series of uncoordinated steps, making it more difficult for the control system to respond.

However, the deformation models used are generally not capable of being linked. In processing control terms, we are where the computer industry was 15 years ago. We have not standardized on the control equivalent of IBM versus Macintosh, we still use different incompatible software, and no one seems to have dreamed of an enterprise-wide Intranet. Fortunately for the computer industry, the increase in performance at reduced cost has meant that the user has been prepared to scrap the computer bought in 1983 and now has a network solution. In fact, those using a PC then are probably on their fourth or fifth machine by now.



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Figure 14. A schematic illustration of an upstream and a downstream process with adaptive control. Less variation in the output of the downstream process is obtained if data can be transferred from upstream.

The steel industry is capital intensive—too capital intensive. It cannot afford to rip out process control systems as better ones come along; the expected lifetime is certainly more than three years. Consequently, I believe we need to have a vision for a through-process model of the future, such that we move towards it. In the case of residual stresses in sheet, described above, this was relatively straightforward. The software industry has moved towards standard packages for FE analysis, so companies along the supply chain can agree on such a package and, then, in a partnering relationship, agree to share the subroutines for, for example, microstructural control.

There has not been such standardization among the manufacturers of rolling mills and their process control systems. In many ways, we are where the PC industry was in 1983, with proprietary closed-architecture black-box systems. Sharing the deformation models, perhaps within a partnering relationship, might be a way forward, and this has some profound implications for the steel industry, which is much more fragmented than for aluminium, in the sense that no single company has double-digit market share.

Improved accuracy of the modelling prediction can be obtained not only by linking the upstream and downstream deformation models, as described above, but also by linking some of the microstructural models going back as far as the as-cast structure. For example, the strain distribution (which could be used to predict microstructure) across the section shown in figure 1b was calculated assuming uniform starting material. Some modelling is done of the segregation pattern across a cast billet. This local compositional variation could be included in the hot deformation model, so that the chemical composition variation across the section, coming from the cast structure, as well as the deformation strain during rolling, is used to predict local properties. Clearly, this is adding complexity, but if local predictions of properties are needed, again their accuracy can be improved.

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The modelling approaches outlined in this paper are mostly macroscopic, describing volumes of material in the range  $0.01-0.1 \text{ m}^3!$  Discontinuities in the structure have only been considered at the grain-size level. Some models describe the deformation behaviour at the quantum mechanical atomistic/molecular level, so it is relevant to ask whether the steel industry needs such models to be further developed. In my view, the answer is ves, although we will probably never model the volume of steel shown in figure 1a using first-principle equations describing each atom, even if computing power allows it. However, one of the arguments presented in this paper is the need to link the models and transfer the data from the upstream to the downstream process. At the microstructural level, a dislocation present in the steel on leaving the hot mill will probably still exist on entering the cold mill. Describing the deformation behaviour using models based on dislocation dynamics, has, at least intuitively, the capability of providing these links and enabling the transfer. Furthermore, we already know that the accuracy of the models currently used generally increases as they become physically based. Consequently, the constitutive equations used will increasingly be from first principles rather than with artificial-intelligencegenerated coefficients. At this point it is not clear in detail how *ab initio* modelling will influence the deformation models used to control a steel mill, but I have every expectation that these control models will benefit from the *ab initio* studies. It might be the next generation of modellers that implements them, but such fundamental work at universities should be encouraged.

The use of process control data to predict the properties of steel can be projected to a vision for the next century. Some device will be automatically put on each piece of steel made. The device will act like a bar code, all the process control data embedded in the manufacture of that piece of steel will be captured, and the customer will have a hand-held device, the equivalent of a bar code reader, attached to a small computer, which will tell him all the specific property information predicted from deformation models. No mechanical testing will be done. The implications of such a vision on the need to link the deformation models are profound.

#### 5. Summary

The deformation models needed by the steel industry in the longer term can be summarized as follows.

- (1) FE analysis software is fairly mature. However, significant advances are to be expected from much better material property data to feed into the models.
- (2) Models for predicting the microstructure after deformation, and, through the microstructure, the mechanical properties, are starting to be successful for metallurgically simple cases, e.g. microstructural evolution during austenitic rolling of C–Mn steels. These models need to be developed further and extended to more complex steels, in particular, to model the transformation from austenite, either after or during the deformation process itself. Adequate models to predict toughness do not yet exist.
- (3) Physically based models need to be incorporated into the process-control systems used to manufacture steel, since they generally lead to better control.

Where the model runs too slowly to do on-line control directly, artificialintelligence systems can be developed, which are fast enough to operate in real time yet allow many of the benefits of the physically based model to be incorporated.

(4) The development of through-process models that allow the flow of knowledge and data through the various process steps will lead to major long-term improvements. Some limited linking of the models at the steel finishing end is already happening. However, the steel industry and its equipment and process control suppliers need to rethink the way the models are developed to ensure that they are capable of migration towards an integrated through-process model in the longer term.

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